EECS 644 HW 7: due 12/4/25

1. For the sequences $x_1(n) = \{2, 2, 1, -1\}$ and $x_2(n) = \{3, 1, 5\}$ compute the circular convolution using DFTs for orders $N = 4$, 5, and 6 (use Matlab and show DFT results and final convolution results). Compare the results with that obtained by linear convolution.

2. From the DFT analysis and synthesis equations, construct (on paper, not with Matlab) the 3×3 DFT and IDFT transform matrices $\mathbf{B}_{\text{DFT}}$ and $\mathbf{B}_{\text{IDFT}}$ which are applied as $\mathbf{X}(k) = \mathbf{B}_{\text{DFT}}^H\, \mathbf{x}(n)$ and $\mathbf{x}(n) = \mathbf{B}_{\text{IDFT}}^H\, \mathbf{X}(k)$ (note the complex-conjugation by using the Hermitian operator). Show that the product of the DFT and IDFT matrices is the identity matrix (*i.e.* $\mathbf{B}_{\text{DFT}}\, \mathbf{B}_{\text{IDFT}} = \mathbf{I}$), thus meaning they are the inverse transformation of one another.

3. For the sequence $x(n) = \{1, 3, 1, -1, -3\}$ apply (by hand) the efficient iterative implementation (slide 11-10) of the Goertzel algorithm to obtain $X(3)$. Check your answer via FFT of $x(n)$ using Matlab.

4. Derive the 9-point Decimation-in-Time FFT (Hint: use radix-3) and sketch the signal flow diagram. Label only the nodes (*i.e.* it is not necessary to label the edges).