1) $x_1(n) = \{2, 2, 1, -1\}$ $x_2(n) = \{3, 1, 5\}$

$$x_1(n) * x_2(n) = \{6, 8, 15, 8, 4, -5\} = x_3(n)$$

N=4

$X_1^{(4)} = \{4, 1-j3, 2, 1+j3\}$

$X_2^{(4)} = \{9, -2-j, 7, -2+j\}$ $\leftarrow$ $x_2(n)$ padded w/ 1 zero

$X_3^{(4)} = X_1^{(4)} \odot X_2^{(4)} = \{36, -5+j5, 14, -5-j5\}$

$x_3^{(4)}(n) = \{10, 3, 15, 8\}$

N=5

$X_1^{(5)} \cong \{4, 2.6-j3.1, 0.4+j0.7, 0.4-j0.7, 2.6+j3.1\}$

$X_2^{(5)} \cong \{9, -0.7-j3.9, 3.7+j4.2, 3.7-j4.2, -0.7+j3.9\}$

$X_3^{(5)} \cong \{36, -13.9-j7.9, -1.6+j4.3, -1.6-j4.3, -13.9+j7.9\}$

$x_3^{(5)}(n) = \{1, 8, 15, 8, 4\}$

N=6

$X_1^{(6)} \cong \{4, 3.5-j2.6, -0.5-j0.9, 2, -0.5+j0.9, 3.5+j2.6\}$

$X_2^{(6)} \cong \{9, 1-j5.2, j3.5, 7, -j3.5, 1+j5.2\}$

$X_3^{(6)} \cong \{36, -10-j20.8, 3-j1.7, 14, 3+j1.7, -10+j20.8\}$

$x_3^{(6)}(n) = \{6, 8, 15, 8, 4, -5)\}$

same as $x_3(n)$ above since

$$N = 6 = N_1 + N_2 - 1$$
$$= 4 + 3 - 1$$

2)

$$B_{DFT} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \\ 1 & e^{j\frac{4\pi}{3}} & e^{j\frac{8\pi}{3}} \end{bmatrix}$$

$$B_{IDFT} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{8\pi}{3}} \end{bmatrix}$$

$$B_{DFT} \cdot B_{IDFT} = \begin{bmatrix} \frac{1}{3}(3) & \frac{1}{3}\left[1 + e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}\right] & \frac{1}{3}\left[1 + e^{-j\frac{4\pi}{3}} + e^{-j\frac{8\pi}{3}}\right] \\ \frac{1}{3}\left[1 + e^{j\frac{2\pi}{3}} + e^{j\frac{4\pi}{3}}\right] & \frac{1}{3}(3) & \frac{1}{3}\left[1 + e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}\right] \\ \frac{1}{3}\left[1 + e^{j\frac{4\pi}{3}} + e^{j\frac{8\pi}{3}}\right] & \frac{1}{3}\left[1 + e^{j\frac{2\pi}{3}} + e^{j\frac{4\pi}{3}}\right] & \frac{1}{3}(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3) $x(n) = \{1, 3, 1, -1, -3\}$

$k = 3$

$N = 5$

$\cos\left(\frac{2\pi 3}{5}\right) \cong -0.81$

$V_3(0) = 1$

$V_3(1) = 2(-0.81)(1) + 3 = 1.38$

$V_3(2) = 2(-0.81)(1.38) - 1 + 1 \cong -2.24$

$V_3(3) = 2(-0.81)(-2.24) - 1.38 - 1 \cong 1.25$

$V_3(4) = 2(-0.81)(1.25) + 2.24 - 3 \cong -2.79$

$V_3(5) = 2(-0.81)(-2.79) - 1.25 + 0 \cong 3.27$

$y_3(5) = 3.27 - e^{-j\frac{2\pi 3}{5}}(-2.79) \cong 1.01 + j1.64$

check by FFT:

$$X \cong \begin{bmatrix} 1 \\ 1 - j6.88 \\ 1 - j1.62 \\ 1 + j1.62 \\ 1 + j6.88 \end{bmatrix} \begin{matrix} \\ \\ \\ \leftarrow X(3) \\ \\ \end{matrix}$$

H)

$$X(k) = \sum_{r=0}^{\frac{N}{3}-1} x(3r)\, W_N^{3rk} + \sum_{r=0}^{\frac{N}{3}-1} x(3r+1)\, W_N^{(3r+1)k} + \sum_{r=0}^{\frac{N}{3}-1} x(3r+2)\, W_N^{(3r+2)k}$$

$k = 0, 1, \dots, 8$

$$= \underbrace{\sum_{r=0}^{\frac{N}{3}-1} x(3r)\, W_{N/3}^{rk}}_{G_1(k)} + W_N^{k} \underbrace{\sum_{r=0}^{\frac{N}{3}-1} x(3r+1)\, W_{N/3}^{rk}}_{G_2(k)} + W_N^{2k} \underbrace{\sum_{r=0}^{\frac{N}{3}-1} x(3r+2)\, W_{N/3}^{rk}}_{G_3(k)}$$

for $k = 0, 1, 2$

$N = 9$