EECS 844 – Fall 2026

Exam 2 Cover page*

Each student is expected to complete the exam individually using only course notes, the book, and technical literature, and without aid from other outside sources.

I assert that I have neither provided help nor accepted help from another student in completing this exam. As such, the work herein is mine and mine alone.

_______________________________________ _________

Signature Date

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Name (printed) Student ID #

*** Attach as cover page to completed exam.**

EECS 844 Exam 2 (Due March 12)

Data sets can be found at http://www.ittc.ku.edu/~sdblunt/844/EECS844_Exam2

Provide: Complete and concise answers to all questions
Matlab code with solutions as appropriate
All solution material (including discussion and figures) for a given problem together (*i.e.* don't put all the plots or code at the end)
Email final Matlab code to me in a .zip file (all together in 1 email)

** All data time sequences are column vectors with **increasing** time index as one traverses down the vector. (you will need to properly orient the data into "snapshots")

1. In the dataset P1.mat are two signals, the input $x(n)$ and desired response $d(n)$ from an unknown system we wish to identify (i.e. system identification). Construct the Wiener filter and determine the resulting MSE for filter lengths of $M = 0, 1, 2, \ldots, 20$. ($M = 0$ means no filter)

a) Plot the MSE (in dB) as a function of length M, using <u>both</u> direct calculation (via average of $|e(n)|^2$) and the analytical MSE (recall the 'error performance surface'). Discuss what you observe. *{Hint: you should observe monotonic reduction in MSE as M increases.}*
b) Plot the magnitude and phase of the (time domain) filter coefficients for the $M = 20$ case.
c) Use the 'freqz' command to plot the frequency response of the $M = 20$ filter. Use the modifier "whole" (see 'help freqz' for details) to plot the entire 2π digital frequency interval.
d) If we assume the unknown system is a MA model, what would you estimate to be length of the unknown MA model and why?

2. Now repeat problem 1 but let $d(n)$ be the <u>input</u> and $x(n)$ be the <u>desired response</u>. Do we learn anything more about the unknown system when the problem is posed this way? Explain what you believe is happening for these two different arrangements. *{Hint: compare the frequency responses.}*

3. Dataset P3.mat contains 100 temporal snapshots (in the columns) from an $M = 18$ element uniform linear array (ULA) with half-wavelength element spacing (so $d = 0.5\lambda$).

a) Plot the MVDR power (spatial) spectrum estimate in dB in terms of spatial angle ϕ. Discuss what you observe (e.g. how many signals do there appear to be?).
b) Plot the non-adaptive (spatial) spectrum estimate defined in Appendix A in terms of ϕ. Describe how this result relates to the adaptive power spectrum.

c) Plot the MVDR beampattern for the spatial directions $\phi = -60°$, $0°$, and $+60°$ (in terms of ϕ), and explain what you observe relative to the MVDR power spectrum from part a).

4. Dataset P4.mat contains the discrete-time signal $x(n)$ for which we wish to isolate the individual frequency components (recall that $f \in [-0.5, +0.5]$). For filter lengths of $M = 10$, 20, 40, and 80 plot the associated MVDR power spectrum for each (in dB). Discuss what you observe.

5. Using data set P4.mat again, implement an $M = 40$ GSC having only a unity gain constraint at frequency $f = +0.25$. See Appendix B.

a) Compare the frequency response (implement same as the beampattern) of the resulting GSC filter with the frequency response of an MVDR filter having the same gain constraint. (Plot in dB)

b) Now use eigenvector constraints to define a broad null over the (normalized) spectral region $f \in [-0.3, -0.1]$. (plot the eigenvalues in dB)

c) Implement the GSC with the same gain constraint and with 0, 5, 10, and 15 eigenvector null constraints. Plot the associated frequency responses and comment on what you observe.

6. Dataset P6.mat contains four signals $x_1(n)$, $x_2(n)$, $x_3(n)$, and $x_4(n)$ that are related through unknown LTI systems, each of which conforms to an MA model of order no more than 30. By examining the relationship between these signals (use tools you have learned so far in this course), determine and sketch the signal flow network, labeling where each signal exists and each branch with the number of significant non-zero coefficients in the associated LTI MA system. Discuss/show how your results were obtained. *{Hint: Think about the number of coefficients that occur when systems are combined in series.}*

Appendix A: Determining a non-adaptive power spectrum

Define the matrix $\mathbf{S}$ comprised of steering vectors "over-sampled" in angle as described in Exam 1. Given the set of snapshots $\mathbf{X} = \left[\mathbf{x}_1\ \mathbf{x}_2\ \cdots\ \mathbf{x}_L\right]$, for each spatial angle ϕ compute the non-adaptive power spectrum as

$$p(\phi) = \frac{1}{L}\sum_{k=1}^{L}\left|\mathbf{s}^H(\phi)\,\mathbf{x}_\ell\right|^2 .$$

Appendix B: Two Matlab techniques to generate an orthogonal complement matrix

For the constraint matrix $\mathbf{C}$ of size $M \times L$:

1) Generate a $M \times (M - L)$ matrix $\mathbf{Z}$ of random complex values.
2) Determine an orthonormal basis for the range of $\mathbf{C}$ as $\mathbf{Q} = orth(\mathbf{C})$ such that $\mathbf{Q}$ spans the same subspace as $\mathbf{C}$.
3) Generate the matrix $\mathbf{P}$ that projects onto the null space of $\mathbf{C}$ as $\mathbf{P} = \mathbf{I} - \mathbf{Q}\mathbf{Q}^H$ (can be done because the columns of $\mathbf{Q}$ are orthonormal).
4) Apply the null projection to the matrix of random values to form an $M \times (M - L)$ orthogonal complement matrix as $\mathbf{C}_a = \mathbf{PZ}$. Normalize each column of $\mathbf{C}_a$ to have unity gain to avoid scale variations.

An alternative approach is to use the last $(M - L)$ left-singular vectors in $\mathbf{U}$ obtained after applying the singular value decomposition (SVD) as $svd(\mathbf{C}) = \mathbf{U\Sigma V}^H$. Of the two approaches, this one is clearly much simpler (and is recommended).